三角函數半角公式

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

證明

1. 由餘弦的兩倍角公式 $\cos 2\alpha = 1 - 2\sin^2\alpha$

   代 $\alpha = \frac{\theta}{2}$, $\quad \cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

   $$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$$
   $$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

2. 由餘弦的兩倍角公式 $\cos 2\alpha = 2\cos^2\alpha - 1$

   代 $\alpha = \frac{\theta}{2}$, $\quad \cos\theta = 2\cos^2\frac{\theta}{2} - 1$

   $$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$$
   $$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

3. 由公式 $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\quad \tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

   $$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

4. 由正弦及餘弦的半角公式

   - $$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}} = \frac{1-\cos\theta}{\sin\theta}$$
   - $$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{\sin\theta}{1+\cos\theta}$$