三角函數兩倍角公式

$$
\begin{aligned}
\sin 2\alpha &= 2\sin\alpha\cos\alpha = 2\sin\alpha\sin(90^\circ+\alpha)\\
\cos 2\alpha &= \cos^2\alpha-\sin^2\alpha = 2\cos^2\alpha-1 = 1-2\sin^2\alpha\\
\tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha}
\end{aligned}
$$

證明

1. 由正弦的兩角和公式 $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$
   (a) $\sin 2\alpha=\sin(\alpha+\alpha)=\sin\alpha\cos\alpha+\sin\alpha\cos\alpha=2\sin\alpha\cos\alpha$
   (b) $$\begin{aligned}\sin 2\alpha &= 2\sin\alpha\cos\alpha = 2\sin\alpha\sin(90^\circ-\alpha)\\ &= 2\sin\alpha\sin(180^\circ-(90^\circ-\alpha))\\ &= 2\sin\alpha\sin(90^\circ+\alpha)\end{aligned}$$

2. 由餘弦的兩角和公式 $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
   (a) $$\begin{aligned}\cos 2\alpha &= \cos(\alpha+\alpha)=\cos\alpha\cos\alpha-\sin\alpha\sin\alpha\\ &= \cos^2\alpha-\sin^2\alpha\end{aligned}$$
   (b) 因 $\sin^2\alpha+\cos^2\alpha=1$
   $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=(1-\sin^2\alpha)-\sin^2\alpha=1-2\sin^2\alpha$
   (c) 同理，$\cos 2\alpha=2\cos^2\alpha-1$

3. 由正切的兩角和公式 $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$
\begin{aligned}
\tan 2\alpha &= \tan(\alpha+\alpha)\\
&= \frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}=\frac{2\tan\alpha}{1-\tan^2\alpha}
\end{aligned}
$$